**Program 1 Report**

**Introduction and instruction how to run the programs**I am turning in two separate C codes, one for Best-First Search (BFS.c) and one for A\* Search (AStar.c). The five initial states for the searches are the same for each code and the initial states are hard coded into the program. The initial state board is represented as a 3x3 integer matrix called *board* that is passed into the *initial* node. Each node holds a 3x3 integer matrix called *state* and calculated f(n) value as integer variable *f*. Nodes also contain node pointers to be used for *reached* linear linked list that keeps track of previously visited nodes and *frontier* priority queue. Nodes for A\* also contains integer variable *d* for keeping track of the depth, which is used as g(n) for f(n) calculations. In the 3x3 integer matrix, integer 9 represents the blank space. The initial state can be changed by changing the hardcoded values assigned to *board* for each state. Heuristics can be changed for each program for all five initial states by changing the value of integer variable *heuristic* in the *main*() function. Once compiled and ran, the programs should output solutions for all five initial states. The number of steps possible are limited to 10,000 for states with no solutions. All provided initial states have solutions.

**Description of the three heuristics**  
The first heuristic, 1h, I used was the number of tiles that are misplaced as described in the book. The blank space did not count as misplaced. Each misplaced tile counted as 1. The second heuristic, 2h, I used was the sum of Manhattan distance of the tiles that are misplaced, not counting the blank, as described in the book. Manhattan distance was measured by how many squares horizontal and vertical the tile has to move to reach its goal state. The third heuristic, 3h, I used was a modified Manhattan distance where diagonal distancing as shortcut was possible.

For example:

|  |  |  |
| --- | --- | --- |
|  | 2 | 3 |
| 5 | 6 | 1 |
| 7 | 8 | 4 |

Tile 1 would be considered to have distance of 2 (shown in blue line) in 3h while it would have the distance of 3 in 2h. I considered using Pythagorean distances but did not want to deal with floating numbers and thought my method would suffice, since it still seem to be admissible as the numbers stayed between those calculated by 1h and 2h. In 1h, tile 1 would just count as 1 tile misplaced as the location of the tile does not matter.   
Again using the example above to demonstrate, reproduced below, calculations are done as follows for the heuristics:

|  |  |  |
| --- | --- | --- |
|  | 2 | 3 |
| 5 | 6 | 1 |
| 7 | 8 | 4 |

1h = 1 + 0 + 0 + 1 + 1 + 1 + 0 + 0 = 4  
2h = 3 + 0 + 0 + 1 + 1 + 3 + 0 + 0 = 8  
3h = 2 + 0 + 0 + 2 + 1 + 1 + 0 + 0 = 6  
**Data  
Best-First Search  
*Heuristic 1*Initial State 1:** 7 4 2 1 6 5 8 3 b  
**Solution Path:**( 7 4 2 1 6 5 8 3 b )->( 7 4 2 1 6 b 8 3 5 )->( 7 4 2 1 b 6 8 3 5 )->( 7 4 2 1 3 6 8 b 5 )->( 7 4 2 1 3 6 b 8 5 )->( 7 4 2 b 3 6 1 8 5 )->( b 4 2 7 3 6 1 8 5 )->( 4 b 2 7 3 6 1 8 5 )->( 4 2 b 7 3 6 1 8 5 )->( 4 2 6 7 3 b 1 8 5 )->( 4 2 6 7 b 3 1 8 5 )->( 4 2 6 b 7 3 1 8 5 )->( b 2 6 4 7 3 1 8 5 )->( 2 b 6 4 7 3 1 8 5 )->( 2 7 6 4 b 3 1 8 5 )->( 2 7 6 4 3 b 1 8 5 )->( 2 7 b 4 3 6 1 8 5 )->( 2 b 7 4 3 6 1 8 5 )->( 2 3 7 4 b 6 1 8 5 )->( 2 3 7 4 6 b 1 8 5 )->( 2 3 b 4 6 7 1 8 5 )->( 2 b 3 4 6 7 1 8 5 )->( 2 6 3 4 b 7 1 8 5 )->( 2 6 3 4 7 b 1 8 5 )->( 2 6 3 4 7 5 1 8 b )->( 2 6 3 4 7 5 1 b 8 )->( 2 6 3 4 b 5 1 7 8 )->( 2 6 3 4 5 b 1 7 8 )->( 2 6 3 4 5 8 1 7 b )->( 2 6 3 4 5 8 1 b 7 )->( 2 6 3 4 b 8 1 5 7 )->( 2 b 3 4 6 8 1 5 7 )->( b 2 3 4 6 8 1 5 7 )->( 4 2 3 b 6 8 1 5 7 )->( 4 2 3 1 6 8 b 5 7 )->( 4 2 3 1 6 8 5 b 7 )->( 4 2 3 1 b 8 5 6 7 )->( 4 2 3 b 1 8 5 6 7 )->( b 2 3 4 1 8 5 6 7 )->( 2 b 3 4 1 8 5 6 7 )->( 2 1 3 4 b 8 5 6 7 )->( 2 1 3 4 6 8 5 b 7 )->( 2 1 3 4 6 8 5 7 b )->( 2 1 3 4 6 b 5 7 8 )->( 2 1 3 4 b 6 5 7 8 )->( 2 1 3 4 7 6 5 b 8 )->( 2 1 3 4 7 6 b 5 8 )->( 2 1 3 b 7 6 4 5 8 )->( b 1 3 2 7 6 4 5 8 )->( 1 b 3 2 7 6 4 5 8 )->( 1 7 3 2 b 6 4 5 8 )->( 1 7 3 b 2 6 4 5 8 )->( b 7 3 1 2 6 4 5 8 )->( 7 b 3 1 2 6 4 5 8 )->( 7 2 3 1 b 6 4 5 8 )->( 7 2 3 1 5 6 4 b 8 )->( 7 2 3 1 5 6 4 8 b )->( 7 2 3 1 5 b 4 8 6 )->( 7 2 3 1 b 5 4 8 6 )->( 7 2 3 b 1 5 4 8 6 )->( b 2 3 7 1 5 4 8 6 )->( 2 b 3 7 1 5 4 8 6 )->( 2 1 3 7 b 5 4 8 6 )->( 2 1 3 7 5 b 4 8 6 )->( 2 1 3 7 5 6 4 8 b )->( 2 1 3 7 5 6 4 b 8 )->( 2 1 3 7 5 6 b 4 8 )->( 2 1 3 b 5 6 7 4 8 )->( b 1 3 2 5 6 7 4 8 )->( 1 b 3 2 5 6 7 4 8 )->( 1 5 3 2 b 6 7 4 8 )->( 1 5 3 b 2 6 7 4 8 )->( 1 5 3 7 2 6 b 4 8 )->( 1 5 3 7 2 6 4 b 8 )->( 1 5 3 7 2 6 4 8 b )->( 1 5 3 7 2 b 4 8 6 )->( 1 5 3 7 b 2 4 8 6 )->( 1 b 3 7 5 2 4 8 6 )->( 1 3 b 7 5 2 4 8 6 )->( 1 3 2 7 5 b 4 8 6 )->( 1 3 2 7 5 6 4 8 b )->( 1 3 2 7 5 6 4 b 8 )->( 1 3 2 7 5 6 b 4 8 )->( 1 3 2 b 5 6 7 4 8 )->( 1 3 2 5 b 6 7 4 8 )->( 1 3 2 5 4 6 7 b 8 )->( 1 3 2 5 4 6 7 8 b )->( 1 3 2 5 4 b 7 8 6 )->( 1 3 b 5 4 2 7 8 6 )->( 1 b 3 5 4 2 7 8 6 )->( 1 4 3 5 b 2 7 8 6 )->( 1 4 3 5 2 b 7 8 6 )->( 1 4 b 5 2 3 7 8 6 )->( 1 b 4 5 2 3 7 8 6 )->( 1 2 4 5 b 3 7 8 6 )->( 1 2 4 b 5 3 7 8 6 )->( b 2 4 1 5 3 7 8 6 )->( 2 b 4 1 5 3 7 8 6 )->( 2 4 b 1 5 3 7 8 6 )->( 2 4 3 1 5 b 7 8 6 )->( 2 4 3 1 5 6 7 8 b )->( 2 4 3 1 5 6 7 b 8 )->( 2 4 3 1 b 6 7 5 8 )->( 2 b 3 1 4 6 7 5 8 )->( b 2 3 1 4 6 7 5 8 )->( 1 2 3 b 4 6 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b )  
**Number of steps:** 1432 steps

**Initial State 2:** 8 1 3 4 b 2 7 6 5  
**Solution Path:**( 8 1 3 4 b 2 7 6 5 )->( 8 1 3 4 6 2 7 b 5 )->( 8 1 3 4 6 2 7 5 b )->( 8 1 3 4 6 b 7 5 2 )->( 8 1 3 4 b 6 7 5 2 )->( 8 1 3 4 5 6 7 b 2 )->( 8 1 3 4 5 6 b 7 2 )->( 8 1 3 b 5 6 4 7 2 )->( b 1 3 8 5 6 4 7 2 )->( 1 b 3 8 5 6 4 7 2 )->( 1 5 3 8 b 6 4 7 2 )->( 1 5 3 8 7 6 4 b 2 )->( 1 5 3 8 7 6 b 4 2 )->( 1 5 3 b 7 6 8 4 2 )->( 1 5 3 7 b 6 8 4 2 )->( 1 5 3 7 4 6 8 b 2 )->( 1 5 3 7 4 6 b 8 2 )->( 1 5 3 b 4 6 7 8 2 )->( 1 5 3 4 b 6 7 8 2 )->( 1 b 3 4 5 6 7 8 2 )->( 1 3 b 4 5 6 7 8 2 )->( 1 3 6 4 5 b 7 8 2 )->( 1 3 6 4 b 5 7 8 2 )->( 1 3 6 4 8 5 7 b 2 )->( 1 3 6 4 8 5 7 2 b )->( 1 3 6 4 8 b 7 2 5 )->( 1 3 b 4 8 6 7 2 5 )->( 1 b 3 4 8 6 7 2 5 )->( 1 8 3 4 b 6 7 2 5 )->( 1 8 3 4 6 b 7 2 5 )->( 1 8 b 4 6 3 7 2 5 )->( 1 b 8 4 6 3 7 2 5 )->( 1 6 8 4 b 3 7 2 5 )->( 1 6 8 4 2 3 7 b 5 )->( 1 6 8 4 2 3 7 5 b )->( 1 6 8 4 2 b 7 5 3 )->( 1 6 b 4 2 8 7 5 3 )->( 1 b 6 4 2 8 7 5 3 )->( 1 2 6 4 b 8 7 5 3 )->( 1 2 6 4 5 8 7 b 3 )->( 1 2 6 4 5 8 7 3 b )->( 1 2 6 4 5 b 7 3 8 )->( 1 2 b 4 5 6 7 3 8 )->( 1 b 2 4 5 6 7 3 8 )->( 1 5 2 4 b 6 7 3 8 )->( 1 5 2 4 3 6 7 b 8 )->( 1 5 2 4 3 6 7 8 b )->( 1 5 2 4 3 b 7 8 6 )->( 1 5 b 4 3 2 7 8 6 )->( 1 b 5 4 3 2 7 8 6 )->( 1 3 5 4 b 2 7 8 6 )->( 1 3 5 4 2 b 7 8 6 )->( 1 3 b 4 2 5 7 8 6 )->( 1 b 3 4 2 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 223 steps

**Initial State 3:** 2 3 8 7 1 4 b 6 5  
**Solution Path:**( 2 3 8 7 1 4 b 6 5 )->( 2 3 8 b 1 4 7 6 5 )->( b 3 8 2 1 4 7 6 5 )->( 3 b 8 2 1 4 7 6 5 )->( 3 1 8 2 b 4 7 6 5 )->( 3 1 8 2 6 4 7 b 5 )->( 3 1 8 2 6 4 7 5 b )->( 3 1 8 2 6 b 7 5 4 )->( 3 1 8 2 b 6 7 5 4 )->( 3 1 8 2 5 6 7 b 4 )->( 3 1 8 2 5 6 b 7 4 )->( 3 1 8 b 5 6 2 7 4 )->( b 1 8 3 5 6 2 7 4 )->( 1 b 8 3 5 6 2 7 4 )->( 1 5 8 3 b 6 2 7 4 )->( 1 5 8 3 7 6 2 b 4 )->( 1 5 8 3 7 6 b 2 4 )->( 1 5 8 b 7 6 3 2 4 )->( 1 5 8 7 b 6 3 2 4 )->( 1 b 8 7 5 6 3 2 4 )->( 1 8 b 7 5 6 3 2 4 )->( 1 8 6 7 5 b 3 2 4 )->( 1 8 6 7 5 4 3 2 b )->( 1 8 6 7 5 4 3 b 2 )->( 1 8 6 7 5 4 b 3 2 )->( 1 8 6 b 5 4 7 3 2 )->( 1 8 6 5 b 4 7 3 2 )->( 1 8 6 5 3 4 7 b 2 )->( 1 8 6 5 3 4 7 2 b )->( 1 8 6 5 3 b 7 2 4 )->( 1 8 b 5 3 6 7 2 4 )->( 1 b 8 5 3 6 7 2 4 )->( 1 3 8 5 b 6 7 2 4 )->( 1 3 8 5 2 6 7 b 4 )->( 1 3 8 5 2 6 7 4 b )->( 1 3 8 5 2 b 7 4 6 )->( 1 3 b 5 2 8 7 4 6 )->( 1 b 3 5 2 8 7 4 6 )->( 1 2 3 5 b 8 7 4 6 )->( 1 2 3 5 4 8 7 b 6 )->( 1 2 3 5 4 8 7 6 b )->( 1 2 3 5 4 b 7 6 8 )->( 1 2 3 5 b 4 7 6 8 )->( 1 2 3 5 6 4 7 b 8 )->( 1 2 3 5 6 4 7 8 b )->( 1 2 3 5 6 b 7 8 4 )->( 1 2 3 5 b 6 7 8 4 )->( 1 2 3 b 5 6 7 8 4 )->( 1 2 3 7 5 6 b 8 4 )->( 1 2 3 7 5 6 8 b 4 )->( 1 2 3 7 b 6 8 5 4 )->( 1 2 3 7 6 b 8 5 4 )->( 1 2 3 7 6 4 8 5 b )->( 1 2 3 7 6 4 8 b 5 )->( 1 2 3 7 6 4 b 8 5 )->( 1 2 3 b 6 4 7 8 5 )->( 1 2 3 6 b 4 7 8 5 )->( 1 2 3 6 8 4 7 b 5 )->( 1 2 3 6 8 4 b 7 5 )->( 1 2 3 b 8 4 6 7 5 )->( 1 2 3 8 b 4 6 7 5 )->( 1 2 3 8 7 4 6 b 5 )->( 1 2 3 8 7 4 6 5 b )->( 1 2 3 8 7 b 6 5 4 )->( 1 2 3 8 b 7 6 5 4 )->( 1 2 3 8 5 7 6 b 4 )->( 1 2 3 8 5 7 6 4 b )->( 1 2 3 8 5 b 6 4 7 )->( 1 2 3 8 b 5 6 4 7 )->( 1 2 3 8 4 5 6 b 7 )->( 1 2 3 8 4 5 b 6 7 )->( 1 2 3 b 4 5 8 6 7 )->( 1 2 3 4 b 5 8 6 7 )->( 1 2 3 4 5 b 8 6 7 )->( 1 2 3 4 5 7 8 6 b )->( 1 2 3 4 5 7 8 b 6 )->( 1 2 3 4 5 7 b 8 6 )->( 1 2 3 b 5 7 4 8 6 )->( 1 2 3 5 b 7 4 8 6 )->( 1 2 3 5 7 b 4 8 6 )->( 1 2 3 5 7 6 4 8 b )->( 1 2 3 5 7 6 4 b 8 )->( 1 2 3 5 b 6 4 7 8 )->( 1 2 3 b 5 6 4 7 8 )->( 1 2 3 4 5 6 b 7 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 323 steps

**Initial State 4:** 7 2 4 5 b 6 8 3 1  
**Solution Path:**( 7 2 4 5 b 6 8 3 1 )->( 7 2 4 5 3 6 8 b 1 )->( 7 2 4 5 3 6 8 1 b )->( 7 2 4 5 3 b 8 1 6 )->( 7 2 4 5 b 3 8 1 6 )->( 7 2 4 b 5 3 8 1 6 )->( b 2 4 7 5 3 8 1 6 )->( 2 b 4 7 5 3 8 1 6 )->( 2 4 b 7 5 3 8 1 6 )->( 2 4 3 7 5 b 8 1 6 )->( 2 4 3 7 5 6 8 1 b )->( 2 4 3 7 5 6 8 b 1 )->( 2 4 3 7 5 6 b 8 1 )->( 2 4 3 b 5 6 7 8 1 )->( b 4 3 2 5 6 7 8 1 )->( 4 b 3 2 5 6 7 8 1 )->( 4 5 3 2 b 6 7 8 1 )->( 4 5 3 2 6 b 7 8 1 )->( 4 5 3 2 6 1 7 8 b )->( 4 5 3 2 6 1 7 b 8 )->( 4 5 3 2 b 1 7 6 8 )->( 4 b 3 2 5 1 7 6 8 )->( b 4 3 2 5 1 7 6 8 )->( 2 4 3 b 5 1 7 6 8 )->( 2 4 3 7 5 1 b 6 8 )->( 2 4 3 7 5 1 6 b 8 )->( 2 4 3 7 5 1 6 8 b )->( 2 4 3 7 5 b 6 8 1 )->( 2 4 3 7 b 5 6 8 1 )->( 2 b 3 7 4 5 6 8 1 )->( b 2 3 7 4 5 6 8 1 )->( 7 2 3 b 4 5 6 8 1 )->( 7 2 3 6 4 5 b 8 1 )->( 7 2 3 6 4 5 8 b 1 )->( 7 2 3 6 b 5 8 4 1 )->( 7 2 3 6 5 b 8 4 1 )->( 7 2 3 6 5 1 8 4 b )->( 7 2 3 6 5 1 8 b 4 )->( 7 2 3 6 5 1 b 8 4 )->( 7 2 3 b 5 1 6 8 4 )->( 7 2 3 5 b 1 6 8 4 )->( 7 2 3 5 1 b 6 8 4 )->( 7 2 3 5 1 4 6 8 b )->( 7 2 3 5 1 4 6 b 8 )->( 7 2 3 5 b 4 6 1 8 )->( 7 2 3 b 5 4 6 1 8 )->( 7 2 3 6 5 4 b 1 8 )->( 7 2 3 6 5 4 1 b 8 )->( 7 2 3 6 5 4 1 8 b )->( 7 2 3 6 5 b 1 8 4 )->( 7 2 3 6 b 5 1 8 4 )->( 7 2 3 b 6 5 1 8 4 )->( b 2 3 7 6 5 1 8 4 )->( 2 b 3 7 6 5 1 8 4 )->( 2 6 3 7 b 5 1 8 4 )->( 2 6 3 7 5 b 1 8 4 )->( 2 6 3 7 5 4 1 8 b )->( 2 6 3 7 5 4 1 b 8 )->( 2 6 3 7 5 4 b 1 8 )->( 2 6 3 b 5 4 7 1 8 )->( b 6 3 2 5 4 7 1 8 )->( 6 b 3 2 5 4 7 1 8 )->( 6 5 3 2 b 4 7 1 8 )->( 6 5 3 2 1 4 7 b 8 )->( 6 5 3 2 1 4 7 8 b )->( 6 5 3 2 1 b 7 8 4 )->( 6 5 3 2 b 1 7 8 4 )->( 6 b 3 2 5 1 7 8 4 )->( 6 3 b 2 5 1 7 8 4 )->( 6 3 1 2 5 b 7 8 4 )->( 6 3 1 2 b 5 7 8 4 )->( 6 b 1 2 3 5 7 8 4 )->( b 6 1 2 3 5 7 8 4 )->( 2 6 1 b 3 5 7 8 4 )->( 2 6 1 3 b 5 7 8 4 )->( 2 6 1 3 5 b 7 8 4 )->( 2 6 b 3 5 1 7 8 4 )->( 2 b 6 3 5 1 7 8 4 )->( b 2 6 3 5 1 7 8 4 )->( 3 2 6 b 5 1 7 8 4 )->( 3 2 6 5 b 1 7 8 4 )->( 3 2 6 5 1 b 7 8 4 )->( 3 2 6 5 1 4 7 8 b )->( 3 2 6 5 1 4 7 b 8 )->( 3 2 6 5 b 4 7 1 8 )->( 3 2 6 b 5 4 7 1 8 )->( 3 2 6 7 5 4 b 1 8 )->( 3 2 6 7 5 4 1 b 8 )->( 3 2 6 7 5 4 1 8 b )->( 3 2 6 7 5 b 1 8 4 )->( 3 2 b 7 5 6 1 8 4 )->( 3 b 2 7 5 6 1 8 4 )->( b 3 2 7 5 6 1 8 4 )->( 7 3 2 b 5 6 1 8 4 )->( 7 3 2 5 b 6 1 8 4 )->( 7 b 2 5 3 6 1 8 4 )->( 7 2 b 5 3 6 1 8 4 )->( 7 2 6 5 3 b 1 8 4 )->( 7 2 6 5 b 3 1 8 4 )->( 7 2 6 b 5 3 1 8 4 )->( 7 2 6 1 5 3 b 8 4 )->( 7 2 6 1 5 3 8 b 4 )->( 7 2 6 1 5 3 8 4 b )->( 7 2 6 1 5 b 8 4 3 )->( 7 2 b 1 5 6 8 4 3 )->( 7 b 2 1 5 6 8 4 3 )->( b 7 2 1 5 6 8 4 3 )->( 1 7 2 b 5 6 8 4 3 )->( 1 7 2 8 5 6 b 4 3 )->( 1 7 2 8 5 6 4 b 3 )->( 1 7 2 8 b 6 4 5 3 )->( 1 b 2 8 7 6 4 5 3 )->( 1 2 b 8 7 6 4 5 3 )->( 1 2 6 8 7 b 4 5 3 )->( 1 2 6 8 7 3 4 5 b )->( 1 2 6 8 7 3 4 b 5 )->( 1 2 6 8 b 3 4 7 5 )->( 1 2 6 b 8 3 4 7 5 )->( 1 2 6 4 8 3 b 7 5 )->( 1 2 6 4 8 3 7 b 5 )->( 1 2 6 4 b 3 7 8 5 )->( 1 2 6 4 3 b 7 8 5 )->( 1 2 6 4 3 5 7 8 b )->( 1 2 6 4 3 5 7 b 8 )->( 1 2 6 4 b 5 7 3 8 )->( 1 2 6 4 5 b 7 3 8 )->( 1 2 b 4 5 6 7 3 8 )->( 1 b 2 4 5 6 7 3 8 )->( 1 5 2 4 b 6 7 3 8 )->( 1 5 2 4 3 6 7 b 8 )->( 1 5 2 4 3 6 7 8 b )->( 1 5 2 4 3 b 7 8 6 )->( 1 5 b 4 3 2 7 8 6 )->( 1 b 5 4 3 2 7 8 6 )->( 1 3 5 4 b 2 7 8 6 )->( 1 3 5 4 2 b 7 8 6 )->( 1 3 b 4 2 5 7 8 6 )->( 1 b 3 4 2 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 626 steps

**Initial State 5:** 5 2 3 7 b 8 6 1 4  
**Solution Path:**( 5 2 3 7 b 8 6 1 4 )->( 5 2 3 7 1 8 6 b 4 )->( 5 2 3 7 1 8 b 6 4 )->( 5 2 3 b 1 8 7 6 4 )->( 5 2 3 1 b 8 7 6 4 )->( 5 2 3 1 6 8 7 b 4 )->( 5 2 3 1 6 8 7 4 b )->( 5 2 3 1 6 b 7 4 8 )->( 5 2 3 1 b 6 7 4 8 )->( 5 2 3 b 1 6 7 4 8 )->( b 2 3 5 1 6 7 4 8 )->( 2 b 3 5 1 6 7 4 8 )->( 2 1 3 5 b 6 7 4 8 )->( 2 1 3 b 5 6 7 4 8 )->( b 1 3 2 5 6 7 4 8 )->( 1 b 3 2 5 6 7 4 8 )->( 1 5 3 2 b 6 7 4 8 )->( 1 5 3 b 2 6 7 4 8 )->( 1 5 3 7 2 6 b 4 8 )->( 1 5 3 7 2 6 4 b 8 )->( 1 5 3 7 2 6 4 8 b )->( 1 5 3 7 2 b 4 8 6 )->( 1 5 3 7 b 2 4 8 6 )->( 1 b 3 7 5 2 4 8 6 )->( 1 3 b 7 5 2 4 8 6 )->( 1 3 2 7 5 b 4 8 6 )->( 1 3 2 7 5 6 4 8 b )->( 1 3 2 7 5 6 4 b 8 )->( 1 3 2 7 5 6 b 4 8 )->( 1 3 2 b 5 6 7 4 8 )->( 1 3 2 5 b 6 7 4 8 )->( 1 3 2 5 4 6 7 b 8 )->( 1 3 2 5 4 6 7 8 b )->( 1 3 2 5 4 b 7 8 6 )->( 1 3 b 5 4 2 7 8 6 )->( 1 b 3 5 4 2 7 8 6 )->( 1 4 3 5 b 2 7 8 6 )->( 1 4 3 5 2 b 7 8 6 )->( 1 4 b 5 2 3 7 8 6 )->( 1 b 4 5 2 3 7 8 6 )->( 1 2 4 5 b 3 7 8 6 )->( 1 2 4 b 5 3 7 8 6 )->( b 2 4 1 5 3 7 8 6 )->( 2 b 4 1 5 3 7 8 6 )->( 2 4 b 1 5 3 7 8 6 )->( 2 4 3 1 5 b 7 8 6 )->( 2 4 3 1 5 6 7 8 b )->( 2 4 3 1 5 6 7 b 8 )->( 2 4 3 1 b 6 7 5 8 )->( 2 b 3 1 4 6 7 5 8 )->( b 2 3 1 4 6 7 5 8 )->( 1 2 3 b 4 6 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 704 steps

**Average number of steps:** 661.6steps ***Heuristic 2:*Initial State 1:** 7 4 2 1 6 5 8 3 b  
**Solution Path:**( 7 4 2 1 6 5 8 3 b )->( 7 4 2 1 6 5 8 b 3 )->( 7 4 2 1 6 5 b 8 3 )->( 7 4 2 b 6 5 1 8 3 )->( b 4 2 7 6 5 1 8 3 )->( 4 b 2 7 6 5 1 8 3 )->( 4 2 b 7 6 5 1 8 3 )->( 4 2 5 7 6 b 1 8 3 )->( 4 2 5 7 b 6 1 8 3 )->( 4 2 5 b 7 6 1 8 3 )->( b 2 5 4 7 6 1 8 3 )->( 2 b 5 4 7 6 1 8 3 )->( 2 5 b 4 7 6 1 8 3 )->( 2 5 6 4 7 b 1 8 3 )->( 2 5 6 4 7 3 1 8 b )->( 2 5 6 4 7 3 1 b 8 )->( 2 5 6 4 b 3 1 7 8 )->( 2 b 6 4 5 3 1 7 8 )->( b 2 6 4 5 3 1 7 8 )->( 4 2 6 b 5 3 1 7 8 )->( 4 2 6 1 5 3 b 7 8 )->( 4 2 6 1 5 3 7 b 8 )->( 4 2 6 1 b 3 7 5 8 )->( 4 2 6 b 1 3 7 5 8 )->( b 2 6 4 1 3 7 5 8 )->( 2 b 6 4 1 3 7 5 8 )->( 2 1 6 4 b 3 7 5 8 )->( 2 1 6 4 3 b 7 5 8 )->( 2 1 b 4 3 6 7 5 8 )->( 2 b 1 4 3 6 7 5 8 )->( 2 3 1 4 b 6 7 5 8 )->( 2 3 1 4 5 6 7 b 8 )->( 2 3 1 4 5 6 7 8 b )->( 2 3 1 4 5 b 7 8 6 )->( 2 3 b 4 5 1 7 8 6 )->( 2 b 3 4 5 1 7 8 6 )->( b 2 3 4 5 1 7 8 6 )->( 4 2 3 b 5 1 7 8 6 )->( 4 2 3 5 b 1 7 8 6 )->( 4 2 3 5 1 b 7 8 6 )->( 4 2 b 5 1 3 7 8 6 )->( 4 b 2 5 1 3 7 8 6 )->( 4 1 2 5 b 3 7 8 6 )->( 4 1 2 b 5 3 7 8 6 )->( b 1 2 4 5 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 167 steps

**Initial State 2:** 8 1 3 4 b 2 7 6 5  
**Solution Path:**( 8 1 3 4 b 2 7 6 5 )->( 8 1 3 4 6 2 7 b 5 )->( 8 1 3 4 6 2 7 5 b )->( 8 1 3 4 6 b 7 5 2 )->( 8 1 3 4 b 6 7 5 2 )->( 8 1 3 4 5 6 7 b 2 )->( 8 1 3 4 5 6 b 7 2 )->( 8 1 3 b 5 6 4 7 2 )->( b 1 3 8 5 6 4 7 2 )->( 1 b 3 8 5 6 4 7 2 )->( 1 5 3 8 b 6 4 7 2 )->( 1 5 3 b 8 6 4 7 2 )->( 1 5 3 4 8 6 b 7 2 )->( 1 5 3 4 8 6 7 b 2 )->( 1 5 3 4 b 6 7 8 2 )->( 1 5 3 4 6 b 7 8 2 )->( 1 5 3 4 6 2 7 8 b )->( 1 5 3 4 6 2 7 b 8 )->( 1 5 3 4 b 2 7 6 8 )->( 1 b 3 4 5 2 7 6 8 )->( 1 3 b 4 5 2 7 6 8 )->( 1 3 2 4 5 b 7 6 8 )->( 1 3 2 4 5 8 7 6 b )->( 1 3 2 4 5 8 7 b 6 )->( 1 3 2 4 b 8 7 5 6 )->( 1 3 2 4 8 b 7 5 6 )->( 1 3 2 4 8 6 7 5 b )->( 1 3 2 4 8 6 7 b 5 )->( 1 3 2 4 b 6 7 8 5 )->( 1 3 2 4 6 b 7 8 5 )->( 1 3 b 4 6 2 7 8 5 )->( 1 b 3 4 6 2 7 8 5 )->( 1 6 3 4 b 2 7 8 5 )->( 1 6 3 4 2 b 7 8 5 )->( 1 6 3 4 2 5 7 8 b )->( 1 6 3 4 2 5 7 b 8 )->( 1 6 3 4 b 5 7 2 8 )->( 1 6 3 4 5 b 7 2 8 )->( 1 6 b 4 5 3 7 2 8 )->( 1 b 6 4 5 3 7 2 8 )->( 1 5 6 4 b 3 7 2 8 )->( 1 5 6 4 2 3 7 b 8 )->( 1 5 6 4 2 3 7 8 b )->( 1 5 6 4 2 b 7 8 3 )->( 1 5 b 4 2 6 7 8 3 )->( 1 b 5 4 2 6 7 8 3 )->( 1 2 5 4 b 6 7 8 3 )->( 1 2 5 4 6 b 7 8 3 )->( 1 2 b 4 6 5 7 8 3 )->( 1 b 2 4 6 5 7 8 3 )->( 1 6 2 4 b 5 7 8 3 )->( 1 6 2 4 5 b 7 8 3 )->( 1 6 2 4 5 3 7 8 b )->( 1 6 2 4 5 3 7 b 8 )->( 1 6 2 4 b 3 7 5 8 )->( 1 b 2 4 6 3 7 5 8 )->( 1 2 b 4 6 3 7 5 8 )->( 1 2 3 4 6 b 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 109 steps

**Initial State 3:** 2 3 8 7 1 4 b 6 5  
**Solution Path:**( 2 3 8 7 1 4 b 6 5 )->( 2 3 8 b 1 4 7 6 5 )->( 2 3 8 1 b 4 7 6 5 )->( 2 3 8 1 6 4 7 b 5 )->( 2 3 8 1 6 4 7 5 b )->( 2 3 8 1 6 b 7 5 4 )->( 2 3 b 1 6 8 7 5 4 )->( 2 b 3 1 6 8 7 5 4 )->( b 2 3 1 6 8 7 5 4 )->( 1 2 3 b 6 8 7 5 4 )->( 1 2 3 6 b 8 7 5 4 )->( 1 2 3 6 5 8 7 b 4 )->( 1 2 3 6 5 8 7 4 b )->( 1 2 3 6 5 b 7 4 8 )->( 1 2 3 6 b 5 7 4 8 )->( 1 2 3 b 6 5 7 4 8 )->( 1 2 3 7 6 5 b 4 8 )->( 1 2 3 7 6 5 4 b 8 )->( 1 2 3 7 6 5 4 8 b )->( 1 2 3 7 6 b 4 8 5 )->( 1 2 3 7 b 6 4 8 5 )->( 1 2 3 b 7 6 4 8 5 )->( 1 2 3 4 7 6 b 8 5 )->( 1 2 3 4 7 6 8 b 5 )->( 1 2 3 4 b 6 8 7 5 )->( 1 2 3 4 6 b 8 7 5 )->( 1 2 3 4 6 5 8 7 b )->( 1 2 3 4 6 5 8 b 7 )->( 1 2 3 4 b 5 8 6 7 )->( 1 2 3 4 5 b 8 6 7 )->( 1 2 3 4 5 7 8 6 b )->( 1 2 3 4 5 7 8 b 6 )->( 1 2 3 4 5 7 b 8 6 )->( 1 2 3 b 5 7 4 8 6 )->( 1 2 3 5 b 7 4 8 6 )->( 1 2 3 5 7 b 4 8 6 )->( 1 2 3 5 7 6 4 8 b )->( 1 2 3 5 7 6 4 b 8 )->( 1 2 3 5 b 6 4 7 8 )->( 1 2 3 b 5 6 4 7 8 )->( 1 2 3 4 5 6 b 7 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 64 steps

**Initial State 4:** 7 2 4 5 b 6 8 3 1  
**Solution Path:**( 7 2 4 5 b 6 8 3 1 )->( 7 2 4 5 3 6 8 b 1 )->( 7 2 4 5 3 6 8 1 b )->( 7 2 4 5 3 b 8 1 6 )->( 7 2 4 5 b 3 8 1 6 )->( 7 2 4 5 1 3 8 b 6 )->( 7 2 4 5 1 3 b 8 6 )->( 7 2 4 b 1 3 5 8 6 )->( 7 2 4 1 b 3 5 8 6 )->( 7 2 4 1 8 3 5 b 6 )->( 7 2 4 1 8 3 b 5 6 )->( 7 2 4 b 8 3 1 5 6 )->( b 2 4 7 8 3 1 5 6 )->( 2 b 4 7 8 3 1 5 6 )->( 2 4 b 7 8 3 1 5 6 )->( 2 4 3 7 8 b 1 5 6 )->( 2 4 3 7 8 6 1 5 b )->( 2 4 3 7 8 6 1 b 5 )->( 2 4 3 7 b 6 1 8 5 )->( 2 b 3 7 4 6 1 8 5 )->( b 2 3 7 4 6 1 8 5 )->( 7 2 3 b 4 6 1 8 5 )->( 7 2 3 1 4 6 b 8 5 )->( 7 2 3 1 4 6 8 b 5 )->( 7 2 3 1 4 6 8 5 b )->( 7 2 3 1 4 b 8 5 6 )->( 7 2 3 1 b 4 8 5 6 )->( 7 2 3 1 5 4 8 b 6 )->( 7 2 3 1 5 4 b 8 6 )->( 7 2 3 b 5 4 1 8 6 )->( b 2 3 7 5 4 1 8 6 )->( 2 b 3 7 5 4 1 8 6 )->( 2 5 3 7 b 4 1 8 6 )->( 2 5 3 7 4 b 1 8 6 )->( 2 5 3 7 4 6 1 8 b )->( 2 5 3 7 4 6 1 b 8 )->( 2 5 3 7 b 6 1 4 8 )->( 2 b 3 7 5 6 1 4 8 )->( b 2 3 7 5 6 1 4 8 )->( 7 2 3 b 5 6 1 4 8 )->( 7 2 3 1 5 6 b 4 8 )->( 7 2 3 1 5 6 4 b 8 )->( 7 2 3 1 b 6 4 5 8 )->( 7 2 3 b 1 6 4 5 8 )->( b 2 3 7 1 6 4 5 8 )->( 2 b 3 7 1 6 4 5 8 )->( 2 1 3 7 b 6 4 5 8 )->( 2 1 3 7 5 6 4 b 8 )->( 2 1 3 7 5 6 b 4 8 )->( 2 1 3 b 5 6 7 4 8 )->( 2 1 3 5 b 6 7 4 8 )->( 2 b 3 5 1 6 7 4 8 )->( b 2 3 5 1 6 7 4 8 )->( 5 2 3 b 1 6 7 4 8 )->( 5 2 3 1 b 6 7 4 8 )->( 5 2 3 1 4 6 7 b 8 )->( 5 2 3 1 4 6 7 8 b )->( 5 2 3 1 4 b 7 8 6 )->( 5 2 b 1 4 3 7 8 6 )->( 5 b 2 1 4 3 7 8 6 )->( b 5 2 1 4 3 7 8 6 )->( 1 5 2 b 4 3 7 8 6 )->( 1 5 2 4 b 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 196 steps

**Initial State 5:** 5 2 3 7 b 8 6 1 4  
**Solution Path:**( 5 2 3 7 b 8 6 1 4 )->( 5 2 3 7 1 8 6 b 4 )->( 5 2 3 7 1 8 b 6 4 )->( 5 2 3 b 1 8 7 6 4 )->( 5 2 3 1 b 8 7 6 4 )->( 5 2 3 1 6 8 7 b 4 )->( 5 2 3 1 6 8 7 4 b )->( 5 2 3 1 6 b 7 4 8 )->( 5 2 3 1 b 6 7 4 8 )->( 5 2 3 1 4 6 7 b 8 )->( 5 2 3 1 4 6 b 7 8 )->( 5 2 3 b 4 6 1 7 8 )->( 5 2 3 4 b 6 1 7 8 )->( 5 b 3 4 2 6 1 7 8 )->( b 5 3 4 2 6 1 7 8 )->( 4 5 3 b 2 6 1 7 8 )->( 4 5 3 1 2 6 b 7 8 )->( 4 5 3 1 2 6 7 b 8 )->( 4 5 3 1 b 6 7 2 8 )->( 4 5 3 b 1 6 7 2 8 )->( b 5 3 4 1 6 7 2 8 )->( 5 b 3 4 1 6 7 2 8 )->( 5 1 3 4 b 6 7 2 8 )->( 5 1 3 4 2 6 7 b 8 )->( 5 1 3 4 2 6 b 7 8 )->( 5 1 3 b 2 6 4 7 8 )->( b 1 3 5 2 6 4 7 8 )->( 1 b 3 5 2 6 4 7 8 )->( 1 2 3 5 b 6 4 7 8 )->( 1 2 3 b 5 6 4 7 8 )->( 1 2 3 4 5 6 b 7 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 144 steps

**Average number of steps:** 136 steps ***Heuristic 3:*Initial State 1:** 7 4 2 1 6 5 8 3 b  
**Solution Path:**( 7 4 2 1 6 5 8 3 b )->( 7 4 2 1 6 b 8 3 5 )->( 7 4 2 1 b 6 8 3 5 )->( 7 4 2 1 3 6 8 b 5 )->( 7 4 2 1 3 6 b 8 5 )->( 7 4 2 b 3 6 1 8 5 )->( b 4 2 7 3 6 1 8 5 )->( 4 b 2 7 3 6 1 8 5 )->( 4 2 b 7 3 6 1 8 5 )->( 4 2 6 7 3 b 1 8 5 )->( 4 2 6 7 b 3 1 8 5 )->( 4 2 6 b 7 3 1 8 5 )->( b 2 6 4 7 3 1 8 5 )->( 2 b 6 4 7 3 1 8 5 )->( 2 6 b 4 7 3 1 8 5 )->( 2 6 3 4 7 b 1 8 5 )->( 2 6 3 4 7 5 1 8 b )->( 2 6 3 4 7 5 1 b 8 )->( 2 6 3 4 b 5 1 7 8 )->( 2 6 3 4 5 b 1 7 8 )->( 2 6 b 4 5 3 1 7 8 )->( 2 b 6 4 5 3 1 7 8 )->( b 2 6 4 5 3 1 7 8 )->( 4 2 6 b 5 3 1 7 8 )->( 4 2 6 1 5 3 b 7 8 )->( 4 2 6 1 5 3 7 b 8 )->( 4 2 6 1 5 3 7 8 b )->( 4 2 6 1 5 b 7 8 3 )->( 4 2 b 1 5 6 7 8 3 )->( 4 b 2 1 5 6 7 8 3 )->( b 4 2 1 5 6 7 8 3 )->( 1 4 2 b 5 6 7 8 3 )->( 1 4 2 5 b 6 7 8 3 )->( 1 b 2 5 4 6 7 8 3 )->( 1 2 b 5 4 6 7 8 3 )->( 1 2 6 5 4 b 7 8 3 )->( 1 2 6 5 4 3 7 8 b )->( 1 2 6 5 4 3 7 b 8 )->( 1 2 6 5 b 3 7 4 8 )->( 1 2 6 5 3 b 7 4 8 )->( 1 2 b 5 3 6 7 4 8 )->( 1 b 2 5 3 6 7 4 8 )->( 1 3 2 5 b 6 7 4 8 )->( 1 3 2 5 4 6 7 b 8 )->( 1 3 2 5 4 6 7 8 b )->( 1 3 2 5 4 b 7 8 6 )->( 1 3 b 5 4 2 7 8 6 )->( 1 b 3 5 4 2 7 8 6 )->( 1 4 3 5 b 2 7 8 6 )->( 1 4 3 5 8 2 7 b 6 )->( 1 4 3 5 8 2 b 7 6 )->( 1 4 3 b 8 2 5 7 6 )->( 1 4 3 8 b 2 5 7 6 )->( 1 4 3 8 2 b 5 7 6 )->( 1 4 3 8 2 6 5 7 b )->( 1 4 3 8 2 6 5 b 7 )->( 1 4 3 8 2 6 b 5 7 )->( 1 4 3 b 2 6 8 5 7 )->( 1 4 3 2 b 6 8 5 7 )->( 1 4 3 2 5 6 8 b 7 )->( 1 4 3 2 5 6 8 7 b )->( 1 4 3 2 5 b 8 7 6 )->( 1 4 3 2 b 5 8 7 6 )->( 1 4 3 b 2 5 8 7 6 )->( 1 4 3 8 2 5 b 7 6 )->( 1 4 3 8 2 5 7 b 6 )->( 1 4 3 8 2 5 7 6 b )->( 1 4 3 8 2 b 7 6 5 )->( 1 4 3 8 b 2 7 6 5 )->( 1 b 3 8 4 2 7 6 5 )->( 1 3 b 8 4 2 7 6 5 )->( 1 3 2 8 4 b 7 6 5 )->( 1 3 2 8 4 5 7 6 b )->( 1 3 2 8 4 5 7 b 6 )->( 1 3 2 8 b 5 7 4 6 )->( 1 3 2 8 5 b 7 4 6 )->( 1 3 b 8 5 2 7 4 6 )->( 1 b 3 8 5 2 7 4 6 )->( 1 5 3 8 b 2 7 4 6 )->( 1 5 3 8 2 b 7 4 6 )->( 1 5 3 8 2 6 7 4 b )->( 1 5 3 8 2 6 7 b 4 )->( 1 5 3 8 2 6 b 7 4 )->( 1 5 3 b 2 6 8 7 4 )->( 1 5 3 2 b 6 8 7 4 )->( 1 b 3 2 5 6 8 7 4 )->( b 1 3 2 5 6 8 7 4 )->( 2 1 3 b 5 6 8 7 4 )->( 2 1 3 8 5 6 b 7 4 )->( 2 1 3 8 5 6 7 b 4 )->( 2 1 3 8 5 6 7 4 b )->( 2 1 3 8 5 b 7 4 6 )->( 2 1 3 8 b 5 7 4 6 )->( 2 1 3 b 8 5 7 4 6 )->( b 1 3 2 8 5 7 4 6 )->( 1 b 3 2 8 5 7 4 6 )->( 1 3 b 2 8 5 7 4 6 )->( 1 3 5 2 8 b 7 4 6 )->( 1 3 5 2 8 6 7 4 b )->( 1 3 5 2 8 6 7 b 4 )->( 1 3 5 2 b 6 7 8 4 )->( 1 b 5 2 3 6 7 8 4 )->( 1 5 b 2 3 6 7 8 4 )->( 1 5 6 2 3 b 7 8 4 )->( 1 5 6 2 b 3 7 8 4 )->( 1 b 6 2 5 3 7 8 4 )->( 1 6 b 2 5 3 7 8 4 )->( 1 6 3 2 5 b 7 8 4 )->( 1 6 3 2 b 5 7 8 4 )->( 1 6 3 2 8 5 7 b 4 )->( 1 6 3 2 8 5 7 4 b )->( 1 6 3 2 8 b 7 4 5 )->( 1 6 3 2 b 8 7 4 5 )->( 1 6 3 b 2 8 7 4 5 )->( 1 6 3 7 2 8 b 4 5 )->( 1 6 3 7 2 8 4 b 5 )->( 1 6 3 7 2 8 4 5 b )->( 1 6 3 7 2 b 4 5 8 )->( 1 6 3 7 b 2 4 5 8 )->( 1 6 3 7 5 2 4 b 8 )->( 1 6 3 7 5 2 b 4 8 )->( 1 6 3 b 5 2 7 4 8 )->( 1 6 3 5 b 2 7 4 8 )->( 1 6 3 5 4 2 7 b 8 )->( 1 6 3 5 4 2 7 8 b )->( 1 6 3 5 4 b 7 8 2 )->( 1 6 b 5 4 3 7 8 2 )->( 1 b 6 5 4 3 7 8 2 )->( 1 4 6 5 b 3 7 8 2 )->( 1 4 6 b 5 3 7 8 2 )->( b 4 6 1 5 3 7 8 2 )->( 4 b 6 1 5 3 7 8 2 )->( 4 6 b 1 5 3 7 8 2 )->( 4 6 3 1 5 b 7 8 2 )->( 4 6 3 1 5 2 7 8 b )->( 4 6 3 1 5 2 7 b 8 )->( 4 6 3 1 b 2 7 5 8 )->( 4 b 3 1 6 2 7 5 8 )->( b 4 3 1 6 2 7 5 8 )->( 1 4 3 b 6 2 7 5 8 )->( 1 4 3 7 6 2 b 5 8 )->( 1 4 3 7 6 2 5 b 8 )->( 1 4 3 7 b 2 5 6 8 )->( 1 4 3 7 2 b 5 6 8 )->( 1 4 3 7 2 8 5 6 b )->( 1 4 3 7 2 8 5 b 6 )->( 1 4 3 7 2 8 b 5 6 )->( 1 4 3 b 2 8 7 5 6 )->( 1 4 3 2 b 8 7 5 6 )->( 1 4 3 2 5 8 7 b 6 )->( 1 4 3 2 5 8 7 6 b )->( 1 4 3 2 5 b 7 6 8 )->( 1 4 3 2 b 5 7 6 8 )->( 1 4 3 2 6 5 7 b 8 )->( 1 4 3 2 6 5 7 8 b )->( 1 4 3 2 6 b 7 8 5 )->( 1 4 3 2 b 6 7 8 5 )->( 1 4 3 b 2 6 7 8 5 )->( b 4 3 1 2 6 7 8 5 )->( 4 b 3 1 2 6 7 8 5 )->( 4 3 b 1 2 6 7 8 5 )->( 4 3 6 1 2 b 7 8 5 )->( 4 3 6 1 b 2 7 8 5 )->( 4 b 6 1 3 2 7 8 5 )->( b 4 6 1 3 2 7 8 5 )->( 1 4 6 b 3 2 7 8 5 )->( 1 4 6 3 b 2 7 8 5 )->( 1 4 6 3 2 b 7 8 5 )->( 1 4 b 3 2 6 7 8 5 )->( 1 b 4 3 2 6 7 8 5 )->( 1 2 4 3 b 6 7 8 5 )->( 1 2 4 b 3 6 7 8 5 )->( b 2 4 1 3 6 7 8 5 )->( 2 b 4 1 3 6 7 8 5 )->( 2 4 b 1 3 6 7 8 5 )->( 2 4 6 1 3 b 7 8 5 )->( 2 4 6 1 b 3 7 8 5 )->( 2 b 6 1 4 3 7 8 5 )->( b 2 6 1 4 3 7 8 5 )->( 1 2 6 b 4 3 7 8 5 )->( 1 2 6 4 b 3 7 8 5 )->( 1 2 6 4 8 3 7 b 5 )->( 1 2 6 4 8 3 7 5 b )->( 1 2 6 4 8 b 7 5 3 )->( 1 2 6 4 b 8 7 5 3 )->( 1 2 6 4 5 8 7 b 3 )->( 1 2 6 4 5 8 7 3 b )->( 1 2 6 4 5 b 7 3 8 )->( 1 2 b 4 5 6 7 3 8 )->( 1 b 2 4 5 6 7 3 8 )->( 1 5 2 4 b 6 7 3 8 )->( 1 5 2 4 3 6 7 b 8 )->( 1 5 2 4 3 6 7 8 b )->( 1 5 2 4 3 b 7 8 6 )->( 1 5 b 4 3 2 7 8 6 )->( 1 b 5 4 3 2 7 8 6 )->( 1 3 5 4 b 2 7 8 6 )->( 1 3 5 4 2 b 7 8 6 )->( 1 3 b 4 2 5 7 8 6 )->( 1 b 3 4 2 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 653 steps

**Initial State 2:** 8 1 3 4 b 2 7 6 5  
**Solution Path:**( 8 1 3 4 b 2 7 6 5 )->( 8 1 3 4 2 b 7 6 5 )->( 8 1 3 4 2 5 7 6 b )->( 8 1 3 4 2 5 7 b 6 )->( 8 1 3 4 b 5 7 2 6 )->( 8 1 3 4 5 b 7 2 6 )->( 8 1 3 4 5 6 7 2 b )->( 8 1 3 4 5 6 7 b 2 )->( 8 1 3 4 5 6 b 7 2 )->( 8 1 3 b 5 6 4 7 2 )->( b 1 3 8 5 6 4 7 2 )->( 1 b 3 8 5 6 4 7 2 )->( 1 5 3 8 b 6 4 7 2 )->( 1 5 3 8 7 6 4 b 2 )->( 1 5 3 8 7 6 b 4 2 )->( 1 5 3 b 7 6 8 4 2 )->( 1 5 3 7 b 6 8 4 2 )->( 1 5 3 7 4 6 8 b 2 )->( 1 5 3 7 4 6 b 8 2 )->( 1 5 3 b 4 6 7 8 2 )->( 1 5 3 4 b 6 7 8 2 )->( 1 5 3 4 6 b 7 8 2 )->( 1 5 3 4 6 2 7 8 b )->( 1 5 3 4 6 2 7 b 8 )->( 1 5 3 4 b 2 7 6 8 )->( 1 5 3 4 2 b 7 6 8 )->( 1 5 b 4 2 3 7 6 8 )->( 1 b 5 4 2 3 7 6 8 )->( 1 2 5 4 b 3 7 6 8 )->( 1 2 5 4 6 3 7 b 8 )->( 1 2 5 4 6 3 7 8 b )->( 1 2 5 4 6 b 7 8 3 )->( 1 2 b 4 6 5 7 8 3 )->( 1 b 2 4 6 5 7 8 3 )->( 1 6 2 4 b 5 7 8 3 )->( 1 6 2 4 5 b 7 8 3 )->( 1 6 2 4 5 3 7 8 b )->( 1 6 2 4 5 3 7 b 8 )->( 1 6 2 4 b 3 7 5 8 )->( 1 b 2 4 6 3 7 5 8 )->( 1 2 b 4 6 3 7 5 8 )->( 1 2 3 4 6 b 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 94 steps

**Initial State 3:** 2 3 8 7 1 4 b 6 5  
**Solution Path:**( 2 3 8 7 1 4 b 6 5 )->( 2 3 8 b 1 4 7 6 5 )->( 2 3 8 1 b 4 7 6 5 )->( 2 3 8 1 4 b 7 6 5 )->( 2 3 b 1 4 8 7 6 5 )->( 2 b 3 1 4 8 7 6 5 )->( b 2 3 1 4 8 7 6 5 )->( 1 2 3 b 4 8 7 6 5 )->( 1 2 3 4 b 8 7 6 5 )->( 1 2 3 4 6 8 7 b 5 )->( 1 2 3 4 6 8 7 5 b )->( 1 2 3 4 6 b 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 16 steps

**Initial State 4:** 7 2 4 5 b 6 8 3 1  
**Solution Path:**( 7 2 4 5 b 6 8 3 1 )->( 7 2 4 5 3 6 8 b 1 )->( 7 2 4 5 3 6 b 8 1 )->( 7 2 4 b 3 6 5 8 1 )->( b 2 4 7 3 6 5 8 1 )->( 2 b 4 7 3 6 5 8 1 )->( 2 4 b 7 3 6 5 8 1 )->( 2 4 6 7 3 b 5 8 1 )->( 2 4 6 7 b 3 5 8 1 )->( 2 b 6 7 4 3 5 8 1 )->( 2 6 b 7 4 3 5 8 1 )->( 2 6 3 7 4 b 5 8 1 )->( 2 6 3 7 4 1 5 8 b )->( 2 6 3 7 4 1 5 b 8 )->( 2 6 3 7 b 1 5 4 8 )->( 2 6 3 7 1 b 5 4 8 )->( 2 6 3 7 1 8 5 4 b )->( 2 6 3 7 1 8 5 b 4 )->( 2 6 3 7 1 8 b 5 4 )->( 2 6 3 b 1 8 7 5 4 )->( 2 6 3 1 b 8 7 5 4 )->( 2 6 3 1 5 8 7 b 4 )->( 2 6 3 1 5 8 7 4 b )->( 2 6 3 1 5 b 7 4 8 )->( 2 6 b 1 5 3 7 4 8 )->( 2 b 6 1 5 3 7 4 8 )->( b 2 6 1 5 3 7 4 8 )->( 1 2 6 b 5 3 7 4 8 )->( 1 2 6 5 b 3 7 4 8 )->( 1 2 6 5 3 b 7 4 8 )->( 1 2 b 5 3 6 7 4 8 )->( 1 b 2 5 3 6 7 4 8 )->( 1 3 2 5 b 6 7 4 8 )->( 1 3 2 5 4 6 7 b 8 )->( 1 3 2 5 4 6 7 8 b )->( 1 3 2 5 4 b 7 8 6 )->( 1 3 b 5 4 2 7 8 6 )->( 1 b 3 5 4 2 7 8 6 )->( 1 4 3 5 b 2 7 8 6 )->( 1 4 3 5 8 2 7 b 6 )->( 1 4 3 5 8 2 b 7 6 )->( 1 4 3 b 8 2 5 7 6 )->( 1 4 3 8 b 2 5 7 6 )->( 1 4 3 8 2 b 5 7 6 )->( 1 4 3 8 2 6 5 7 b )->( 1 4 3 8 2 6 5 b 7 )->( 1 4 3 8 2 6 b 5 7 )->( 1 4 3 b 2 6 8 5 7 )->( 1 4 3 2 b 6 8 5 7 )->( 1 4 3 2 5 6 8 b 7 )->( 1 4 3 2 5 6 8 7 b )->( 1 4 3 2 5 b 8 7 6 )->( 1 4 3 2 b 5 8 7 6 )->( 1 4 3 b 2 5 8 7 6 )->( 1 4 3 8 2 5 b 7 6 )->( 1 4 3 8 2 5 7 b 6 )->( 1 4 3 8 2 5 7 6 b )->( 1 4 3 8 2 b 7 6 5 )->( 1 4 3 8 b 2 7 6 5 )->( 1 b 3 8 4 2 7 6 5 )->( 1 3 b 8 4 2 7 6 5 )->( 1 3 2 8 4 b 7 6 5 )->( 1 3 2 8 4 5 7 6 b )->( 1 3 2 8 4 5 7 b 6 )->( 1 3 2 8 b 5 7 4 6 )->( 1 3 2 8 5 b 7 4 6 )->( 1 3 b 8 5 2 7 4 6 )->( 1 b 3 8 5 2 7 4 6 )->( 1 5 3 8 b 2 7 4 6 )->( 1 5 3 8 2 b 7 4 6 )->( 1 5 3 8 2 6 7 4 b )->( 1 5 3 8 2 6 7 b 4 )->( 1 5 3 8 2 6 b 7 4 )->( 1 5 3 b 2 6 8 7 4 )->( 1 5 3 2 b 6 8 7 4 )->( 1 b 3 2 5 6 8 7 4 )->( b 1 3 2 5 6 8 7 4 )->( 2 1 3 b 5 6 8 7 4 )->( 2 1 3 8 5 6 b 7 4 )->( 2 1 3 8 5 6 7 b 4 )->( 2 1 3 8 5 6 7 4 b )->( 2 1 3 8 5 b 7 4 6 )->( 2 1 3 8 b 5 7 4 6 )->( 2 1 3 b 8 5 7 4 6 )->( b 1 3 2 8 5 7 4 6 )->( 1 b 3 2 8 5 7 4 6 )->( 1 3 b 2 8 5 7 4 6 )->( 1 3 5 2 8 b 7 4 6 )->( 1 3 5 2 8 6 7 4 b )->( 1 3 5 2 8 6 7 b 4 )->( 1 3 5 2 b 6 7 8 4 )->( 1 b 5 2 3 6 7 8 4 )->( 1 5 b 2 3 6 7 8 4 )->( 1 5 6 2 3 b 7 8 4 )->( 1 5 6 2 b 3 7 8 4 )->( 1 b 6 2 5 3 7 8 4 )->( 1 6 b 2 5 3 7 8 4 )->( 1 6 3 2 5 b 7 8 4 )->( 1 6 3 2 b 5 7 8 4 )->( 1 6 3 2 8 5 7 b 4 )->( 1 6 3 2 8 5 7 4 b )->( 1 6 3 2 8 b 7 4 5 )->( 1 6 3 2 b 8 7 4 5 )->( 1 6 3 b 2 8 7 4 5 )->( 1 6 3 7 2 8 b 4 5 )->( 1 6 3 7 2 8 4 b 5 )->( 1 6 3 7 2 8 4 5 b )->( 1 6 3 7 2 b 4 5 8 )->( 1 6 3 7 b 2 4 5 8 )->( 1 6 3 7 5 2 4 b 8 )->( 1 6 3 7 5 2 b 4 8 )->( 1 6 3 b 5 2 7 4 8 )->( 1 6 3 5 b 2 7 4 8 )->( 1 6 3 5 4 2 7 b 8 )->( 1 6 3 5 4 2 7 8 b )->( 1 6 3 5 4 b 7 8 2 )->( 1 6 b 5 4 3 7 8 2 )->( 1 b 6 5 4 3 7 8 2 )->( 1 4 6 5 b 3 7 8 2 )->( 1 4 6 b 5 3 7 8 2 )->( b 4 6 1 5 3 7 8 2 )->( 4 b 6 1 5 3 7 8 2 )->( 4 6 b 1 5 3 7 8 2 )->( 4 6 3 1 5 b 7 8 2 )->( 4 6 3 1 5 2 7 8 b )->( 4 6 3 1 5 2 7 b 8 )->( 4 6 3 1 b 2 7 5 8 )->( 4 b 3 1 6 2 7 5 8 )->( b 4 3 1 6 2 7 5 8 )->( 1 4 3 b 6 2 7 5 8 )->( 1 4 3 7 6 2 b 5 8 )->( 1 4 3 7 6 2 5 b 8 )->( 1 4 3 7 b 2 5 6 8 )->( 1 4 3 7 2 b 5 6 8 )->( 1 4 3 7 2 8 5 6 b )->( 1 4 3 7 2 8 5 b 6 )->( 1 4 3 7 2 8 b 5 6 )->( 1 4 3 b 2 8 7 5 6 )->( 1 4 3 2 b 8 7 5 6 )->( 1 4 3 2 5 8 7 b 6 )->( 1 4 3 2 5 8 7 6 b )->( 1 4 3 2 5 b 7 6 8 )->( 1 4 3 2 b 5 7 6 8 )->( 1 4 3 2 6 5 7 b 8 )->( 1 4 3 2 6 5 7 8 b )->( 1 4 3 2 6 b 7 8 5 )->( 1 4 3 2 b 6 7 8 5 )->( 1 4 3 b 2 6 7 8 5 )->( b 4 3 1 2 6 7 8 5 )->( 4 b 3 1 2 6 7 8 5 )->( 4 3 b 1 2 6 7 8 5 )->( 4 3 6 1 2 b 7 8 5 )->( 4 3 6 1 b 2 7 8 5 )->( 4 b 6 1 3 2 7 8 5 )->( b 4 6 1 3 2 7 8 5 )->( 1 4 6 b 3 2 7 8 5 )->( 1 4 6 3 b 2 7 8 5 )->( 1 4 6 3 2 b 7 8 5 )->( 1 4 b 3 2 6 7 8 5 )->( 1 b 4 3 2 6 7 8 5 )->( 1 2 4 3 b 6 7 8 5 )->( 1 2 4 b 3 6 7 8 5 )->( b 2 4 1 3 6 7 8 5 )->( 2 b 4 1 3 6 7 8 5 )->( 2 4 b 1 3 6 7 8 5 )->( 2 4 6 1 3 b 7 8 5 )->( 2 4 6 1 b 3 7 8 5 )->( 2 b 6 1 4 3 7 8 5 )->( b 2 6 1 4 3 7 8 5 )->( 1 2 6 b 4 3 7 8 5 )->( 1 2 6 4 b 3 7 8 5 )->( 1 2 6 4 8 3 7 b 5 )->( 1 2 6 4 8 3 7 5 b )->( 1 2 6 4 8 b 7 5 3 )->( 1 2 6 4 b 8 7 5 3 )->( 1 2 6 4 5 8 7 b 3 )->( 1 2 6 4 5 8 7 3 b )->( 1 2 6 4 5 b 7 3 8 )->( 1 2 b 4 5 6 7 3 8 )->( 1 b 2 4 5 6 7 3 8 )->( 1 5 2 4 b 6 7 3 8 )->( 1 5 2 4 3 6 7 b 8 )->( 1 5 2 4 3 6 7 8 b )->( 1 5 2 4 3 b 7 8 6 )->( 1 5 b 4 3 2 7 8 6 )->( 1 b 5 4 3 2 7 8 6 )->( 1 3 5 4 b 2 7 8 6 )->( 1 3 5 4 2 b 7 8 6 )->( 1 3 b 4 2 5 7 8 6 )->( 1 b 3 4 2 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 715 steps

**Initial State 5:** 5 2 3 7 b 8 6 1 4  
**Solution Path:**( 5 2 3 7 b 8 6 1 4 )->( 5 2 3 7 1 8 6 b 4 )->( 5 2 3 7 1 8 b 6 4 )->( 5 2 3 b 1 8 7 6 4 )->( 5 2 3 1 b 8 7 6 4 )->( 5 2 3 1 6 8 7 b 4 )->( 5 2 3 1 6 8 7 4 b )->( 5 2 3 1 6 b 7 4 8 )->( 5 2 3 1 b 6 7 4 8 )->( 5 2 3 1 4 6 7 b 8 )->( 5 2 3 1 4 6 7 8 b )->( 5 2 3 1 4 b 7 8 6 )->( 5 2 b 1 4 3 7 8 6 )->( 5 b 2 1 4 3 7 8 6 )->( 5 4 2 1 b 3 7 8 6 )->( 5 4 2 b 1 3 7 8 6 )->( b 4 2 5 1 3 7 8 6 )->( 4 b 2 5 1 3 7 8 6 )->( 4 2 b 5 1 3 7 8 6 )->( 4 2 3 5 1 b 7 8 6 )->( 4 2 3 5 1 6 7 8 b )->( 4 2 3 5 1 6 7 b 8 )->( 4 2 3 5 1 6 b 7 8 )->( 4 2 3 b 1 6 5 7 8 )->( b 2 3 4 1 6 5 7 8 )->( 2 b 3 4 1 6 5 7 8 )->( 2 1 3 4 b 6 5 7 8 )->( 2 1 3 4 7 6 5 b 8 )->( 2 1 3 4 7 6 b 5 8 )->( 2 1 3 b 7 6 4 5 8 )->( b 1 3 2 7 6 4 5 8 )->( 1 b 3 2 7 6 4 5 8 )->( 1 7 3 2 b 6 4 5 8 )->( 1 7 3 2 5 6 4 b 8 )->( 1 7 3 2 5 6 4 8 b )->( 1 7 3 2 5 b 4 8 6 )->( 1 7 3 2 b 5 4 8 6 )->( 1 b 3 2 7 5 4 8 6 )->( 1 3 b 2 7 5 4 8 6 )->( 1 3 5 2 7 b 4 8 6 )->( 1 3 5 2 7 6 4 8 b )->( 1 3 5 2 7 6 4 b 8 )->( 1 3 5 2 b 6 4 7 8 )->( 1 3 5 b 2 6 4 7 8 )->( 1 3 5 4 2 6 b 7 8 )->( 1 3 5 4 2 6 7 b 8 )->( 1 3 5 4 2 6 7 8 b )->( 1 3 5 4 2 b 7 8 6 )->( 1 3 b 4 2 5 7 8 6 )->( 1 b 3 4 2 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 360 steps

**Average number of steps:** 367.6 steps  
**A\* Search**  
***Heuristic 1:*   
Initial State 1:** 7 4 2 1 6 5 8 3 b  
**Solution Path:**( 7 4 2 1 6 5 8 3 b )->( 7 4 2 1 6 5 8 b 3 )->( 7 4 2 1 b 5 8 6 3 )->( 7 4 2 b 1 5 8 6 3 )->( b 4 2 7 1 5 8 6 3 )->( 4 b 2 7 1 5 8 6 3 )->( 4 1 2 7 b 5 8 6 3 )->( 4 1 2 7 5 b 8 6 3 )->( 4 1 2 7 5 3 8 6 b )->( 4 1 2 7 5 3 8 b 6 )->( 4 1 2 7 5 3 b 8 6 )->( 4 1 2 b 5 3 7 8 6 )->( b 1 2 4 5 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 380 steps

**Initial State 2:** 8 1 3 4 b 2 7 6 5  
**Solution Path:**( 8 1 3 4 b 2 7 6 5 )->( 8 1 3 4 2 b 7 6 5 )->( 8 1 3 4 2 5 7 6 b )->( 8 1 3 4 2 5 7 b 6 )->( 8 1 3 4 2 5 b 7 6 )->( 8 1 3 b 2 5 4 7 6 )->( b 1 3 8 2 5 4 7 6 )->( 1 b 3 8 2 5 4 7 6 )->( 1 2 3 8 b 5 4 7 6 )->( 1 2 3 b 8 5 4 7 6 )->( 1 2 3 4 8 5 b 7 6 )->( 1 2 3 4 8 5 7 b 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b )  
**Number of steps:** 260 steps

**Initial State 3:** 2 3 8 7 1 4 b 6 5  
**Solution Path:**( 2 3 8 7 1 4 b 6 5 )->( 2 3 8 b 1 4 7 6 5 )->( 2 3 8 1 b 4 7 6 5 )->( 2 3 8 1 4 b 7 6 5 )->( 2 3 b 1 4 8 7 6 5 )->( 2 b 3 1 4 8 7 6 5 )->( b 2 3 1 4 8 7 6 5 )->( 1 2 3 b 4 8 7 6 5 )->( 1 2 3 4 b 8 7 6 5 )->( 1 2 3 4 6 8 7 b 5 )->( 1 2 3 4 6 8 7 5 b )->( 1 2 3 4 6 b 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 163 steps

**Initial State 4:** 7 2 4 5 b 6 8 3 1  
**Solution Path:**( 7 2 4 5 b 6 8 3 1 )->( 7 2 4 5 3 6 8 b 1 )->( 7 2 4 5 3 6 8 1 b )->( 7 2 4 5 3 b 8 1 6 )->( 7 2 4 5 b 3 8 1 6 )->( 7 2 4 b 5 3 8 1 6 )->( b 2 4 7 5 3 8 1 6 )->( 2 b 4 7 5 3 8 1 6 )->( 2 4 b 7 5 3 8 1 6 )->( 2 4 3 7 5 b 8 1 6 )->( 2 4 3 7 b 5 8 1 6 )->( 2 4 3 7 1 5 8 b 6 )->( 2 4 3 7 1 5 b 8 6 )->( 2 4 3 b 1 5 7 8 6 )->( 2 4 3 1 b 5 7 8 6 )->( 2 b 3 1 4 5 7 8 6 )->( b 2 3 1 4 5 7 8 6 )->( 1 2 3 b 4 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 2585 steps

**Initial State 5:** 5 2 3 7 b 8 6 1 4  
**Solution Path:**( 5 2 3 7 b 8 6 1 4 )->( 5 2 3 7 1 8 6 b 4 )->( 5 2 3 7 1 8 b 6 4 )->( 5 2 3 b 1 8 7 6 4 )->( 5 2 3 1 b 8 7 6 4 )->( 5 2 3 1 8 b 7 6 4 )->( 5 2 3 1 8 4 7 6 b )->( 5 2 3 1 8 4 7 b 6 )->( 5 2 3 1 b 4 7 8 6 )->( 5 2 3 1 4 b 7 8 6 )->( 5 2 b 1 4 3 7 8 6 )->( 5 b 2 1 4 3 7 8 6 )->( b 5 2 1 4 3 7 8 6 )->( 1 5 2 b 4 3 7 8 6 )->( 1 5 2 4 b 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 1045 steps

**Average number of steps:** 886.6 steps ***Heuristic 2:*Initial State 1:** 7 4 2 1 6 5 8 3 b  
**Solution Path:**( 7 4 2 1 6 5 8 3 b )->( 7 4 2 1 6 5 8 b 3 )->( 7 4 2 1 b 5 8 6 3 )->( 7 4 2 b 1 5 8 6 3 )->( b 4 2 7 1 5 8 6 3 )->( 4 b 2 7 1 5 8 6 3 )->( 4 1 2 7 b 5 8 6 3 )->( 4 1 2 7 5 b 8 6 3 )->( 4 1 2 7 5 3 8 6 b )->( 4 1 2 7 5 3 8 b 6 )->( 4 1 2 7 5 3 b 8 6 )->( 4 1 2 b 5 3 7 8 6 )->( b 1 2 4 5 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 100 steps

**Initial State 2:** 8 1 3 4 b 2 7 6 5  
**Solution Path:**( 8 1 3 4 b 2 7 6 5 )->( 8 1 3 4 2 b 7 6 5 )->( 8 1 3 4 2 5 7 6 b )->( 8 1 3 4 2 5 7 b 6 )->( 8 1 3 4 2 5 b 7 6 )->( 8 1 3 b 2 5 4 7 6 )->( b 1 3 8 2 5 4 7 6 )->( 1 b 3 8 2 5 4 7 6 )->( 1 2 3 8 b 5 4 7 6 )->( 1 2 3 b 8 5 4 7 6 )->( 1 2 3 4 8 5 b 7 6 )->( 1 2 3 4 8 5 7 b 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 69 steps

**Initial State 3:** 2 3 8 7 1 4 b 6 5  
**Solution Path:**( 2 3 8 7 1 4 b 6 5 )->( 2 3 8 b 1 4 7 6 5 )->( 2 3 8 1 b 4 7 6 5 )->( 2 3 8 1 4 b 7 6 5 )->( 2 3 b 1 4 8 7 6 5 )->( 2 b 3 1 4 8 7 6 5 )->( b 2 3 1 4 8 7 6 5 )->( 1 2 3 b 4 8 7 6 5 )->( 1 2 3 4 b 8 7 6 5 )->( 1 2 3 4 6 8 7 b 5 )->( 1 2 3 4 6 8 7 5 b )->( 1 2 3 4 6 b 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 19 steps

**Initial State 4:** 7 2 4 5 b 6 8 3 1  
**Solution Path:**( 7 2 4 5 b 6 8 3 1 )->( 7 2 4 5 3 6 8 b 1 )->( 7 2 4 5 3 6 8 1 b )->( 7 2 4 5 3 b 8 1 6 )->( 7 2 4 5 b 3 8 1 6 )->( 7 2 4 b 5 3 8 1 6 )->( b 2 4 7 5 3 8 1 6 )->( 2 b 4 7 5 3 8 1 6 )->( 2 4 b 7 5 3 8 1 6 )->( 2 4 3 7 5 b 8 1 6 )->( 2 4 3 7 b 5 8 1 6 )->( 2 4 3 7 1 5 8 b 6 )->( 2 4 3 7 1 5 b 8 6 )->( 2 4 3 b 1 5 7 8 6 )->( 2 4 3 1 b 5 7 8 6 )->( 2 b 3 1 4 5 7 8 6 )->( b 2 3 1 4 5 7 8 6 )->( 1 2 3 b 4 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 174 steps

**Initial State 5:** 5 2 3 7 b 8 6 1 4  
**Solution Path:**( 5 2 3 7 b 8 6 1 4 )->( 5 2 3 7 1 8 6 b 4 )->( 5 2 3 7 1 8 b 6 4 )->( 5 2 3 b 1 8 7 6 4 )->( 5 2 3 1 b 8 7 6 4 )->( 5 2 3 1 8 b 7 6 4 )->( 5 2 3 1 8 4 7 6 b )->( 5 2 3 1 8 4 7 b 6 )->( 5 2 3 1 b 4 7 8 6 )->( 5 2 3 1 4 b 7 8 6 )->( 5 2 b 1 4 3 7 8 6 )->( 5 b 2 1 4 3 7 8 6 )->( b 5 2 1 4 3 7 8 6 )->( 1 5 2 b 4 3 7 8 6 )->( 1 5 2 4 b 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b )  
**Number of steps:** 181 steps

**Average number of steps:** 108.6 steps ***Heuristic 3:*  
Initial State 1:** 7 4 2 1 6 5 8 3 b  
**Solution Path:**( 7 4 2 1 6 5 8 3 b )->( 7 4 2 1 6 5 8 b 3 )->( 7 4 2 1 b 5 8 6 3 )->( 7 4 2 b 1 5 8 6 3 )->( b 4 2 7 1 5 8 6 3 )->( 4 b 2 7 1 5 8 6 3 )->( 4 1 2 7 b 5 8 6 3 )->( 4 1 2 7 5 b 8 6 3 )->( 4 1 2 7 5 3 8 6 b )->( 4 1 2 7 5 3 8 b 6 )->( 4 1 2 7 5 3 b 8 6 )->( 4 1 2 b 5 3 7 8 6 )->( b 1 2 4 5 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b )  
**Number of steps:** 151 steps **Initial State 2:** 8 1 3 4 b 2 7 6 5  
**Solution Path:**( 8 1 3 4 b 2 7 6 5 )->( 8 1 3 4 2 b 7 6 5 )->( 8 1 3 4 2 5 7 6 b )->( 8 1 3 4 2 5 7 b 6 )->( 8 1 3 4 2 5 b 7 6 )->( 8 1 3 b 2 5 4 7 6 )->( b 1 3 8 2 5 4 7 6 )->( 1 b 3 8 2 5 4 7 6 )->( 1 2 3 8 b 5 4 7 6 )->( 1 2 3 b 8 5 4 7 6 )->( 1 2 3 4 8 5 b 7 6 )->( 1 2 3 4 8 5 7 b 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b )  
**Number of steps:** 120 steps

**Initial State 3:** 2 3 8 7 1 4 b 6 5  
**Solution Path:**( 2 3 8 7 1 4 b 6 5 )->( 2 3 8 b 1 4 7 6 5 )->( 2 3 8 1 b 4 7 6 5 )->( 2 3 8 1 4 b 7 6 5 )->( 2 3 b 1 4 8 7 6 5 )->( 2 b 3 1 4 8 7 6 5 )->( b 2 3 1 4 8 7 6 5 )->( 1 2 3 b 4 8 7 6 5 )->( 1 2 3 4 b 8 7 6 5 )->( 1 2 3 4 6 8 7 b 5 )->( 1 2 3 4 6 8 7 5 b )->( 1 2 3 4 6 b 7 5 8 )->( 1 2 3 4 b 6 7 5 8 )->( 1 2 3 4 5 6 7 b 8 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 39 steps

**Initial State 4:** 7 2 4 5 b 6 8 3 1  
**Solution Path:**( 7 2 4 5 b 6 8 3 1 )->( 7 2 4 5 3 6 8 b 1 )->( 7 2 4 5 3 6 8 1 b )->( 7 2 4 5 3 b 8 1 6 )->( 7 2 4 5 b 3 8 1 6 )->( 7 2 4 b 5 3 8 1 6 )->( b 2 4 7 5 3 8 1 6 )->( 2 b 4 7 5 3 8 1 6 )->( 2 4 b 7 5 3 8 1 6 )->( 2 4 3 7 5 b 8 1 6 )->( 2 4 3 7 b 5 8 1 6 )->( 2 4 3 7 1 5 8 b 6 )->( 2 4 3 7 1 5 b 8 6 )->( 2 4 3 b 1 5 7 8 6 )->( 2 4 3 1 b 5 7 8 6 )->( 2 b 3 1 4 5 7 8 6 )->( b 2 3 1 4 5 7 8 6 )->( 1 2 3 b 4 5 7 8 6 )->( 1 2 3 4 b 5 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 883 steps

**Initial State 5:** 5 2 3 7 b 8 6 1 4  
**Solution Path:**( 5 2 3 7 b 8 6 1 4 )->( 5 2 3 7 1 8 6 b 4 )->( 5 2 3 7 1 8 b 6 4 )->( 5 2 3 b 1 8 7 6 4 )->( 5 2 3 1 b 8 7 6 4 )->( 5 2 3 1 8 b 7 6 4 )->( 5 2 3 1 8 4 7 6 b )->( 5 2 3 1 8 4 7 b 6 )->( 5 2 3 1 b 4 7 8 6 )->( 5 2 3 1 4 b 7 8 6 )->( 5 2 b 1 4 3 7 8 6 )->( 5 b 2 1 4 3 7 8 6 )->( b 5 2 1 4 3 7 8 6 )->( 1 5 2 b 4 3 7 8 6 )->( 1 5 2 4 b 3 7 8 6 )->( 1 b 2 4 5 3 7 8 6 )->( 1 2 b 4 5 3 7 8 6 )->( 1 2 3 4 5 b 7 8 6 )->( 1 2 3 4 5 6 7 8 b ) **Number of steps:** 403 steps

**Average number of steps:** 319.2 steps  
  
**Conclusion**  
A\* search produced optimal solution paths for all three heuristics for every initial state while Best-First-Search (BFS) varied widely in solution paths depending on the heuristics used. For A\* search, initial state 1’s optimal path had 17 moves, initial state 2’s optimal path had 15 moves, initial state 3’s optimal path had 15 moves, initial state 4’s optimal path had 21 moves, and initial state 5’s optimal path had 19 moves no matter which heuristics was used. However, the steps it took to get to the optimal path varied, with heuristics 2 being the fastest for all initial states. For BFS search, initial state 1 had the shortest solution path and steps with heuristics 1, initial state 2 had the shortest solution path and steps with heuristics 3, initial state 3 had the shortest solution path and steps with heuristics 3, initial state 4 had the shortest solution path and steps with heuristics 2, and initial state 5 had the shortest solution and steps path with heuristics 2. In general, heuristic 1, which counted misplaced tiles, took worst average steps for both A\* and BFS searches. Heuristic 3, which is the modified Manhattan distance with diagonals, took the second most average steps for both A\* and BFS searches. Heuristic 2, the Manhattan distance, performed the best average steps for both A\* and BFS searches. Heuristic 1 produced on average 661.6 steps for BFS and 886.6 steps for A\*. Therefore, for heuristics 1, A\* search took longer average steps to find the solution path than BFS. Looking at individual data, BFS found the solutions for initial states 2,3, and 5 faster than A\*. Heuristic 2 produced on average 136 steps for BFS and 108.6 steps for A\* searches. Except for the initial state 5, A\* performed better for all other initial states than BFS using heuristics 2. Heuristic 3 produced on average 367.6 steps for BFS and 319.2 steps for A\*. However, looking at individual data, A\* took longer steps than BFS in all initial states except initial state 1.

In conclusion, whether BFS and A\* finds a solution path faster seems to be dependent on the heuristics used and on the initial state of the puzzle. BFS may not produce the optimal path but may be faster at finding a solution than A\* in some situations depending on heuristics and the initial state of the puzzle. BFS only came close once to finding the optimal solution path in my test. However, A\* will always find the optimal solution path even if it takes greater number of steps. Manhattan distance seems to be the best heuristic to use for both searches in comparison to counting misplaced tiles or modified Manhattan distance, and counting misplaced tiles seem to be the worst out of the three. This seems to indicate the more information the search has of the state (exactly how off each tile is through Manhattan distance v. just the number of tiles misplaced), higher the performance.